

On The Prediction of The Statistical Model

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Abstract

The number of quark flavours n_f , has been derived to be equal to 6 in agreement with the experimental observations. The colour factor ratio $\frac{C_A}{C_F}$ has also been computed and is found to be in exact agreement with the corresponding QCD prediction. Further, the e^+e^- pair annihilation cross section into hadrons at very high energy is found to be a function only of the fractal dimension of the hadron.

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I. Introduction:

The statistical model, since its very inception three decades ago, has undergone various applications with success [1, 2, 3]. The model has a deep rooted concept of a relation between quark matter and the space embedded in it and has explained and predicted a number of interesting and bizzare features [4, 5, 6]. The assignment of the (asymptotic) fractal dimension $D = \frac{9}{2}$ to a hadron has been the crucial factor in analysing the various properties of hadron. Most recently [7], we have studied the colours of quarks as new degrees of freedom and the number of colours equal to 3 has been derived. Further, the fractional nature of the electric charges of quarks has also been suggested. In the current investigation, we have derived the number of quark flavours n_f , the colour factor ratio $\frac{T_F}{C_F}$. The e^+e^- pair annihilation cross section into hadrons at high energy has been studied.

II. Theory:

To deduce the number of quark flavours n_f and to estimate the magnitude of the well known QCD colour factor ratio $\frac{T_F}{C_F}$, we are to study the production of hadrons in the final state initiated by a gluon producing gluons and quarks in the intermediate stages corresponding to the processes 1 and 2 respectively. In the intermediate stages, each energetic gluon or quark creates a jet of hadrons in the final state. However, gluon jets are broader than those in the case of quarks as the gluons can emit bremsstrahlung themselves and this is more pronounced because of greater colour charge. With the increase of the energy of the quarks in the quark jet, the number of quark flavours grows to its maximum value n_f and there would be a close similarity between quark and gluon jets.

As usual, we denote [8] the relative strengths of the splitting probabilities with C_A and T_F for gluon splitting into two gluons and two quarks respectively. If the final state is not differentiated with respect to its flavour content, an additional factor n_f has to be taken into account as the gluon can split into n_f quark flavours. Both the gluon and quark jets in processes 1 and 2 respectively produce hadrons, each having the fractal dimension D as suggested by the model. As fractal dimension is a geometrical property of metric transformations, it remains invariant under such a transformation [9]. Consequently, the two spaces corre-

sponding to the quark and gluon jets at $Q^2 \rightarrow \alpha$ where Q^2 is the square of the characteristic energy scale, would behave like equivalent metricspaces. Hence the spaces would be topologically equivalent i.e. homeomorphic. Unlike isotopy, the two diagrams corresponding to the aforesaid two processes would thus become homeomorphic as it would depend on the diagrams themselves and not on their disposition in space. Hence we come across the vertex V_1 representing the three gluon coupling corresponding to the process 1 and the vertex V_2 representing the coupling of the gluon with two quarks corresponding to the process 2. Therefore, the vertex V_2 at high energy limit would be mapped into the vertex V_1 . Hence the vertex factor $n_f T_F$ in the limit $Q^2 \rightarrow \alpha$ at V_2 would approach the vertex factor C_A at V_1 . Therefore, we have

$$Lt. Q^2 \rightarrow \alpha n_f T_F \rightarrow C_A \quad (1)$$

It is relevant to assert that we have also derived in the framework of the model[7], the magnitude of the colour factor $C_A = 3$, from the fractal dimension of hadron, without any reference to the experimental findings. Using $C_A = 3$ directly in (1), we have $n_f = 6$ since $T_F = \frac{1}{2}$. Further, from (1), we get

$$Lt. Q^2 \rightarrow \alpha n_f \frac{T_F}{C_F} = \frac{C_A}{C_F} \frac{9}{4} \quad (2)$$

where we have used our previously derived value of $\frac{C_A}{C_F}$ from fractal dimension[5]. With our estimate of $n_f = 6$ as an input in (2), we get

$$\frac{T_F}{C_F} = \frac{3}{8} \quad (3)$$

in exact agreement with the corresponding QCD prediction. It is worth mentioning that the aforesaid value of $\frac{T_F}{C_F} = \frac{3}{8}$ in (3) has been obtained from the fractal properties of hadron suggested by model.

It is well-known that the cross section $\sigma_{e^+e^-} \rightarrow \text{hadrons}$ coincides with the cross section $\sigma_{e^+e^-} \rightarrow q \bar{q}$ at very high energies from quark hadron duality and that σ for e^+e^- annihilation

into hadronic final states has the form[8]

$$\sigma = f(\alpha_s C_F, \frac{C_A}{C_F}, n_f \frac{T_F}{C_F}) \quad (4)$$

This is, it is a function of the quark gluon coupling constant α_s in addition to the other colour factors and n_f .

The cross section for the process $e^+e^- \rightarrow \text{hadron}$ is dominated by processes leading to the production of $q\bar{q}$ pairs followed by a strong interaction fragmentation process which converts the high energy $q\bar{q}$ pair into jets of hadrons. As $Q^2 \rightarrow \alpha$, we have $\alpha_s \rightarrow 0$ and $n_f \frac{T_F}{C_F}$ becomes equal to $\frac{C_A}{C_F} = \frac{D}{2} = \frac{9}{4}$. Therefore, we arrive at

$$Lt.Q^2 \rightarrow \alpha \sigma = f(\frac{D}{2}) \quad (5)$$

Thus the quark hadron duality in conjunction with the model suggests that the asymptotic e^+e^- pair annihilation cross section becomes a function of the fractal dimension of hadron.

III. Summary:

one of the most interesting results of the current investigation is the prediction of the number of quark flavours n_f equal to 6 without any reference to the experimental findings. The colour factor ratio $\frac{T_F}{C_F}$ derived is in exact agreement with the corresponding QCD prediction and the asymptotic e^+e^- pair annihilation cross section is found to depend only on the fractal dimension of hadron.

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